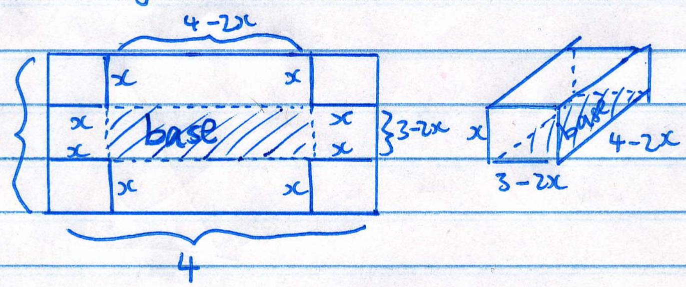
Section 4.4 11/15/12

#26. a. Squares with side length x are cut out of each corner of a rectangular piece of cardboard measuring 3 ft by 4 ft. The resulting piece of cardboard is folded into a box without a lid. Find the volume of the largest box that can be formed in this way.

~~Try to~~ Draw a diagram:



objective function to maximize is volume V:

$$V(x) = x(4-2x)(3-2x)$$

Constraints: $0 < x < 1.5$ (gives $V > 0$)

Aside: $3-2x > 0$ to have sidelength positive

$$3 > 2x$$

$$\frac{3}{2} > x$$

Strategy: find $V'(x)$, solve $V'(x)=0$, figure out if solutions make sense.

How to get $V'(x)$?

Ans: multiply out first $\leftarrow$ $V(x) = x(12-8x-6x-4x)$

$$V(x) = 12x - 14x^2 + 4x^3$$

Solve $V'(x) = 0$

$$12 - 28x + 12x^2 = 0$$

$$4(3 - 7x + 3x^2) = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 3 \cdot 3}}{2 \cdot 3}$$

$$x = \frac{7 \pm \sqrt{13}}{6}$$

$x = \frac{7+\sqrt{13}}{6}$ and $x = \frac{7-\sqrt{13}}{6}$

note:

$\sqrt{9} < \sqrt{13} < \sqrt{16}$

$3 < \sqrt{13} < 4$